

Boğaziçi MATH COLLOQUIUM

The Riemann Zeta Function on the Critical Line

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Abstract:

The Riemann zeta function is defined by the expression $\zeta(s) = \sum_{n \geq 1} \frac{1}{n^s}$ for $s \in \mathbb{C}$ with $\Re s > 1$ and is then meromorphically continued to the whole complex plane $\mathbb{C}$. The vertical line $\{s \in \mathbb{C} : \Re s = 1/2\}$ is called the critical line for the zeta function, which is the place where nontrivial solutions of $\zeta(s) = 0$ is conjectured to lie according to the Riemann hypothesis.

The zeta function has been one of the most important objects of study for analytic number theorists due to its connection to the distribution of prime numbers and recently also became the focus of AI-driven research. I will discuss the recent works by Claude AI, Levent Alpöge and Ralph Furman and by Youness Lamzouri on the proportion of nontrivial zeros that are on the critical line or are simple. I will also review what is known about the value distribution of the zeta function and mention my recent joint work with Steven M. Gonek and Hugh L. Montgomery on a conjecture of K. Ramachandra on the density of values $\{\zeta(1/2 + it) : t \in \mathbb{R}\}$ in the complex plane.

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